MTH 508/608: Introduction to Differentiable manifolds and Lie Groups
Semester 1, 2024-25

General information

Instructor: Dr. Kashyap Rajeevsarathy
Office: AB1 (Infinity building) - 314
E-mail: kashyap@iiserb.ac.in
Venue: AB1 (Infinity building) - 308
Google classroom: dt3jcqs
Schedule: SLOT I (in Time Table)

Day	Timing
Tuesday	3 pm - 4 pm
Thursday	3 pm - 4 pm
Friday	3 pm - 4 pm

Course webpage: Click here.

Course structure

Topics

- *Differentiable manifolds:* definition and examples, differentiable functions, existence of partitions of unity, tangent vectors and tangent space at a point, tangent bundle, differential of a smooth map, inverse function theorem, implicit function theorem, immersions, submanifolds, submersions, Sard's theorem, and the Whitney embedding theorem.

- *Vector fields:* vector fields, statement of the existence theorem for ordinary differential equations, one parameter and local one-parameter groups acting on a manifold, the Lie derivative and the Lie algebra of vector fields, distributions, and the Frobenius theorem.

- *Lie groups:* definition and examples, action of a Lie group on a manifold, definition of Lie algebra, the exponential map, Lie subgroups and closed subgroups, and homogeneous manifolds: definition and examples.

- *Tensor fields and differential forms:* cotangent vectors and the cotangent space at a point, cotangent bundle, covector fields or 1-forms on a manifold, tensors on a vector space, tensor product, symmetric and alternating tensors, the exterior algebra, tensor fields and differential forms on a manifold, and the exterior algebra on a manifold.

- *Integration:* orientation of a manifold, a quick review of Riemann integration in Euclidean spaces, differentiable simplex in a manifold, singular chains, integration of forms over singular chains in a manifold, manifolds with boundary, integration of n-forms over regular domains in an oriented manifold of dimension n, Stokes theorem, definition of de Rham cohomology of a manifold, statement of de Rham theorem, and the Poincare lemma.

Suggested references

1. J. Lee, *Introduction to smooth manifolds*, Springer, 2002.

2. W. Boothby, *An Introduction to differentiable manifolds and Riemannian geometry*, Academic Press, 2002.

3. F. Warner, *Foundations of differentiable manifolds and Lie groups, Springer*, GTM 94, 1983.

4. M. Spivak, *A comprehensive introduction todifferential geometry, Vol. 1*, Publish or Perish, 1999.

Course policies

Classes

- As this is an advanced course, it would be in the best interest of the students to have offline lectures during scheduled class hours.
- However, if the need arises to have online lectures for an extended period of time, please make note of the following points:
  - Reading material and prerecorded video lectures will be posted at the Google Classroom portal every week. It is your responsibility to check the portal for any updates (from my end) and come prepared for the interactive sessions.
  - There will be live interaction sessions every week during the scheduled lecture hours. These sessions will be primarily aimed at enhancing your conceptual understanding of the reading material and video lectures posted during the week. Therefore, you are advised to effectively use these sessions to clarify your doubts on the topics being covered.
  - If you face any difficulties in participating in the live interaction sessions due to limitations in internet connectivity, data availability (or coverage) or technology, please contact me immediately. I will try my best to provide additional help or make alternative arrangements for you. Note that all live sessions will be recorded and posted in the Google Classroom for the benefit of the students with poor internet/data connectivity.

Continuous assessments

Continuous assessment will carry 30% weightage in your final grade as per the current academic policy (available here). This component of your grade will be computed based on two continuous assessment sub-components, namely homework assignments and quizzes, which will each carry a weightage of 15%.

Assignments

- Practice problem sets will be posted every other week. **It is highly imperative that you try solving these problems on your own, as your homework assignments will constitute select problems from these sets.**
- **Up to four homework assignments will be given during the course of the semester that you would have to turn in. Your top two performances in these assignments will together count towards one continuous assessment component.**
- The problems to be turned in and the due dates will be posted on the Google Classroom. So it is your responsibility to regularly check the Google Classroom for any updates.
- If you must miss the due date (for genuine reasons), try turning in your assignment in advance, or write to me seeking an extension.
- If physical submission is not possible, your solutions should be turned in via email either as a typed document or as a scanned softcopy of handwritten solutions.
- Problems written should be legible and must clearly indicate the steps used to arrive at the solution.
- While you are encouraged to share and discuss ideas with your classmates, I would strongly caution you against copying solutions verbatim from your classmate/friend. **Please be warned that:**
  - **(a) Assignments with nearly identical solutions will not be graded.**
  - **(b) If there is evidence that even a part of an assignment is copied (or plagiarized), the entire assignment will be given a score of zero.**

Quizzes

- **Up to two quizzes may be administered during the course of the semester and your best performance in these quizzes will count towards one continuous assessment component.**

- These quizzes will be given in the regular classrooms. However, if the need arises, quizzes and exams may be hosted on any online proctoring platform endorsed by the Institute. In such situations, students may be required to take the quiz while sitting in front of their computer webcams/mobile phone cameras during the entire duration of the examination.
- The schedules and syllabuses for the quizzes with be announced in the Google Classroom.

Mid-semester and end-semester exams

- **In adherence with the current academic policy (available here), mid-semester and end-semester examinations will carry weights of 30% and 40%, respectively, in your final grade.**
- These exams will be administered in the officially assigned classrooms. However, if the need arise, the exams may be hosted on any online proctoring platform endorsed by the Institute. In such situations, students may be required to take the quiz while sitting in front of their computer webcams/mobile phone cameras during the entire duration of the examination.
- The final exam will be comprehensive with possibly more emphasis on the topics that are covered after the mid-semester exam.
- When graded exams are returned, please check them carefully for any grading errors. All grading issues should be brought to my attention as soon as possible. Note that test scores are not renegotiable after final grades are submitted.
- Do not make travel plans that might prevent you from taking any scheduled exam. If you have a verifiable reason why you cannot be present at an exam, you must contact me in advance to make an alternative arrangement.

General policies concerning assessment

- Books, notes, or electronic devices of any kind are strictly prohibited while taking tests (exams and quizzes). It is your ethical (and moral) responsibility to exercise honesty and integrity while taking them.
- When graded tests are returned, please check them carefully for any grading errors. All grading issues should be brought to my attention as soon as possible. Note that your scores are not renegotiable after the final grades are submitted.
- Do not make plans that might prevent you from taking any scheduled exam or quiz. If you have a justifiable reason for missing a scheduled test, you must contact me in advance to make an alternative arrangement.
- Strong disciplinary action will be initiated against students indulging in academic malpractices (or misconduct) during quizzes (or exams) which include any form of cheating, impersonation, copying, plagiarism, etc., as per the prevailing academic norms of the Institute available at: Circular - Disciplinary actions for various acts of academic malpractices.

Grading scheme

A total of 100 percentage points will be distributed as follows:

Component	**Weightage**
Continuous assessment	30%
Mid-semester examination	30%
Final examination	40%